

Topological order in the vortex-glass phase of high-temperature superconductors

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The stability of a vortex glass phase with quasi-long-range positional order is examined for a disordered layered superconductor. The role of topological defects is investigated using a scaling argument supplemented by a variational calculation. The results indicate that topological order is preserved for some range of parameters in the vortex glass phase. The stability regime is given in terms of a simple Lindemann-like criterion and is consistent with recent experiments. [S0163-1829(97)07501-2]

It is well known that the Abrikosov flux lattice in a type-II superconductor is unstable to point disorder beyond the Larkin length.¹ The nature of the flux array at larger scales has been a subject of intense studies.² It has been conjectured that the flux array is *collectively pinned*, forming a vortex glass (VG) phase³⁻⁶ with zero linear resistivity at low temperatures. This conjecture is supported by a number of experiments on disordered samples of high- T_c superconductors,⁷⁻¹⁰ where a continuous transition to a phase with zero linear resistivity was found upon cooling. On the other hand, Bitter-decoration,¹¹ neutron-scattering,¹² and μ SR (Ref. 13) experiments on weakly disordered samples have all indicated some *long-range order* of the flux array, a characteristic usually incompatible with a glass. A common interpretation for the observation of a flux *lattice* is a cross-over effect due to the large Larkin lengths in weakly disordered samples. In this article, we investigate a different possibility, that the flux array may maintain its positional long-range order *much beyond* the Larkin length, in spite of pinning by point disorders.

Such a possibility is indeed realized in a model of a *dislocation-free* flux line array in random media.^{5,14} This model is very similar to the randomly pinned charge-density waves and the random-field XY model which have been studied extensively in the past decades.¹⁴⁻¹⁷ A variety of approximate methods have been used to obtain the conclusion that point disorders lead to a glass phase with only logarithmic fluctuations in the transverse displacement of the flux array. This implies the existence of quasi-long-range positional order in the glass phase of the dislocation-free flux array.¹⁴

As recently suggested,¹⁴ such a topologically ordered glass may actually exist as a *stable* thermodynamic phase for some range of parameters in the cuprate superconductors. Related numerical studies^{18,19} of the random field XY model and a layered model superconductor further supported this scenario. However, the issue of spontaneous formation of topological defects (i.e., dislocation loops) involves compli-

cated interplay between elasticity and disorders, and has so far not been addressed *quantitatively*. In this article, we investigate this issue using a model of flux lines confined in the planes of a layered superconductor. Our model allows for the formation of dislocation loops and is amenable to analytic studies. We first present a scaling argument which yields suppression of large dislocation loops at finite fugacities. This result is then supplemented by a variational calculation, from which we obtain a Lindemann-like criterion giving the size of the stability regime for the topologically ordered VG. Finally, we generalize our argument to the usual experimental situation of flux lines perpendicular to the layers, and compare to experimental findings.

We start with a strongly layered impure superconductor in a *parallel* magnetic field. The superconducting layers provide a sufficiently strong confining potential for the (Josephson-like) vortex lines which exist in the interlayer spacing. We shall exclude the possibility of the lines crossing the superconducting layers. [For fields parallel to the *ab* planes of the Bi compound, typical vortex kink energies are of the order $10^3(1 - T/T_c)^\circ K$.] This amounts to limiting the vortex displacement field from two components in an isotropic sample to one component (i.e., parallel to the layers). For simplicity we shall focus on the *dilute* limit where the inter-vortex spacings l_\perp (interlayer), l_\parallel (intralayer) exceed the magnetic penetration depths λ_{ab} , λ_c respectively. The implications of our results on the dense limit are straightforward and will be discussed below.

A well-established analytic description for a *single* layer of vortex lines (for length scales exceeding l_\parallel) is given by the Hamiltonian³

$$\beta\mathcal{H}_{2D}[\phi_j, W_j] = \int_{\mathbf{r}_\parallel} \left\{ \frac{K}{2} (\nabla_\parallel \phi_j)^2 - W_j [\phi_j(\mathbf{r}_\parallel), \mathbf{r}_\parallel] \right\}, \quad (1)$$

where $\phi_j(\mathbf{r}_\parallel)$ describes the in-plane displacement of the vortex lines in the j th layer and K is an (isotropized) in-plane elastic constant. Pinning effects due to point disorder are

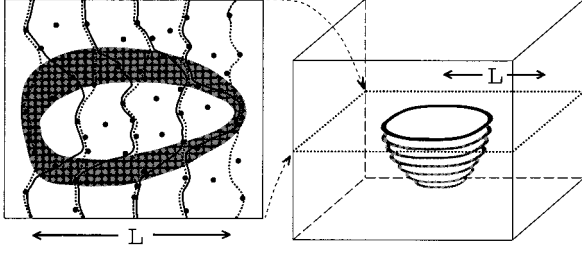


FIG. 1. Left: Elastic rearrangement of each planar flux array can be represented by a number of vortex loops (shaded region). Right: Aligned vortex loops in successive layers form a vortex sheet. The boundary of the sheet (dark loop on top) is a dislocation loop.

described by the random potential $W_j[\phi_j(\mathbf{r}_\parallel), \mathbf{r}_\parallel]$, with the second moment $\overline{W_j[\phi, \mathbf{r}]W_j[0, 0]} = g_0^2 \cos[\phi] \delta^2(\mathbf{r})$, where the overbar denotes disorder average, g_0 characterizes the (bare) strength of the random potentials, and the cosine captures the discrete nature of the vortex lines.⁵ With many layers stacked next to each other, the Hamiltonian for the whole system is

$$\beta\mathcal{H} = \sum_j \left\{ \beta\mathcal{H}_{2D}[\phi_j, W_j] + \int_{\mathbf{r}_\parallel} V_j[\phi_{j+1}(\mathbf{r}_\parallel) - \phi_j(\mathbf{r}_\parallel)] \right\}, \quad (2)$$

$$V_j[\phi] = -\mu \cos[\phi],$$

where $\overline{W_j W_{j'}} = \delta_{j,j'} \overline{W_j W_j}$ since the bare random potentials in different layers are uncorrelated. The interaction $V[\phi]$ in Eq. (2) can be regarded as the repulsive magnetic interaction energy between the lowest harmonics of density fluctuations between vortex lines in “adjacent” layers, a valid approximation in the dilute limit.²⁰ The coupling constant μ is related to the shear modulus of the flux line lattice. The main feature of this model is that it goes beyond the elastic approximation, as it allows for dislocation loops between adjacent layers (see Fig. 1).

In what follows, we first study the phase diagram of the system in terms of the parameters μ and K for a fixed strength of disorder. If the vortex layers are uncoupled, i.e., $\mu = 0$, then each layer undergoes separately a glass transition at a critical value $K_c = 1/4\pi$,^{3,16} with a vanishing linear resistivity in the low-temperature phase ($K > K_c$) and ohmic behavior in the high temperature phase ($K < K_c$).²¹ The physical range of interest corresponds to $K \gg K_c$.

The limit of very weak coupling ($\mu \ll 1$) may be studied using perturbation theory; it is straightforward to find that a weak coupling is irrelevant at large scales. In the limit of very large coupling $\mu \rightarrow \infty$, the interaction potential $V[\phi]$ may be replaced by the quadratic form, $\mu\phi^2/2$, which describes an elastic (i.e., dislocation-free) coupling in the direction perpendicular to the layers. This is just the anisotropic, one-component version of the VG considered previously in Refs. 5, 14, and 15. After introducing a continuum description in terms of $\mathbf{r} = (\mathbf{r}_\parallel, r_\perp)$ and rescaling $r_\perp = (j l_\perp) \sqrt{\mu l_\perp^2 / K}$, we get an isotropic three-dimensional (3D) elastic Hamiltonian

$$\beta\mathcal{H}_{3D} = \int d^3\mathbf{r} \left\{ \frac{\gamma}{2} (\nabla\phi)^2 - W[\phi(\mathbf{r}), \mathbf{r}] \right\} \quad (3)$$

with an effective elastic constant $\gamma = \sqrt{\mu K}$ and a random potential $W[\phi(\mathbf{r}), \mathbf{r}]$ with $W[\phi, \mathbf{r}]W[0, 0] = g_0^2 \cos[\phi] \delta^3(\mathbf{r})$, where $g^2 = g_0^2 \sqrt{\mu/K}$. From various methods including position-space RG,¹⁵ a Flory-type argument,⁵ functional RG,¹⁴ and a variational Ansatz,^{14,17} one finds that the system (3) forms a glass phase with

$$\overline{[\phi(\mathbf{r}) - \phi(\mathbf{r}')]^2}_{3D} = 2A \ln(|\mathbf{r} - \mathbf{r}'|/\ell) \quad (4)$$

beyond the positional correlation length $\ell \sim \gamma^2/g^2$, with A being an universal number of $O(1)$.²² The logarithmic fluctuation in displacement leads to quasi-long-range order and an (algebraic) Bragg peak at reciprocal lattice vector $2\pi/l_\parallel$. This phase was referred to as the “Bragg glass.”¹⁴ In 3D, there is a large elastic energy cost, of the order $(\gamma/2) \int d^3\mathbf{r} (\nabla\phi)^2_{3D} \sim \gamma L$, for logarithmic fluctuations in a volume of the order L^3 . This energy is compensated by the disorder energy gained from the anomalous displacement of the flux array. Thus $\Delta E \sim \gamma L$ gives the order of variation in free energy for configurations which differ in ϕ by $O(\sqrt{\ln(L/\ell)})$. Note also that the elastic Bragg glass has a nonzero response to shear.

Given the above properties of the Bragg glass, which exists so far only in the unphysical limit $\mu \rightarrow \infty$, our first task is to determine whether it can persist at a finite μ , i.e., whether the system is stable to the spontaneous formation of dislocation loops on length scales much larger than the correlation length ℓ . To investigate this possibility, we divide the system into two halves (within which the layers are elastically coupled) and allow dislocation loops to form in the contact plane, say between the j_0 th and (j_0+1) th layers. Analytically, this is implemented by using the following interaction energy in Eq. (2):

$$V'_j[\phi] = \frac{\mu}{2} \phi^2 (1 - \delta_{j,j_0}) - \mu' \cos[\phi] \delta_{j,j_0} \quad (5)$$

with $\mu' \approx \mu \gg 1$ approximating $V_j[\phi]$. It is useful to consider first arbitrary values of μ' . If the two halves of the system are decoupled (i.e., $\mu' = 0$), then each forms a Bragg glass, and the configuration of the flux array in each half is individually optimized. But if the two halves are tied together (i.e., $\mu' \rightarrow \infty$), then the constraint across the contact plane forces a complete reoptimization of the flux array, resulting in a higher free energy for each half. Since the constraint amounts to changing the boundary condition $\phi_{j_0}(\mathbf{r}_\parallel)$ of the half-systems, by $O[\sqrt{\ln(L/\ell)}]$ according to Eq. (4), the typical free energy increase in each half due to the constraint is given by $\Delta E \sim \gamma L$.

Observe that the difference between the optimal configuration of each half system resulting from the constraint at j_0 can be described by a collection of “vortex sheets” such as the one depicted in Fig. 1. A dislocation loop, which describes phase mismatches across the contact plane, is just the boundary of a vortex sheet at the contact plane. The argument above thus shows that the disorder energy gained from the proliferation of dislocations loops (i.e., complete decoupling) at the contact plane is ΔE . Consequently, the disorder energy gained from the formation of a single dislocation loop is $E_{\text{dis}} \leq \Delta E$. Assuming scaling of this energy, $E_{\text{dis}} \sim \gamma' L^\omega$, it follows that $\omega \leq 1$.

We next consider the interaction energy cost due to the formation of dislocation loops. In the large (but finite) μ' limit of interest, it is sufficient to focus on large L , the stability of a *single* optimally configured dislocation loop of extent $L \gg \ell$ at the contact plane of the two (elastic) half systems (Fig. 1). The energy cost of the core of the dislocation loop due to the interlayer interaction is extensive. For a stretched circular loop of linear size L , we expect $E_{\text{core}} \sim \mu' \ell L$. Here, ℓ appears as the ‘‘thickness’’ of the loop because the flux array is elastically coupled at smaller scales.²³ More generally, if we allow the dislocation loop to take on fractal shapes, say with the total loop length scaling as L^D for $L \gg \ell$ ($D \geq 1$ being the fractal dimension), then the core energy becomes $E_{\text{core}} \sim \mu' \ell^{2-D} L^D$.

The existence or not of dislocation loops can now be determined by comparing this core energy with the gain in disorder energy, $E_{\text{dis}} \sim \gamma' L^\omega$. The value of the exponent ω depends on the structure of the dislocation loop we allow, i.e., on the fractal dimension D . We expect that the upper bound ($\omega = 1$) may only be reached if the structure of the associated vortex sheet is similar to those that arise when the coupling at the contact plane is changed from $\mu' = \infty$ to $\mu' = 0$. The structure of the latter can be deduced as follows: Denote the difference in the configuration before/after the change in μ' by $\varphi(\mathbf{r})$. The vortex sheets are then the equal- φ contours of $\varphi(\mathbf{r})$, and the associated dislocation loops are the contours of $\varphi(\mathbf{r}_{\parallel}, r_{\perp} = j_0 l_{\perp})$. The relationship between a rough ‘‘landscape’’ and the fractal geometry of its contours have recently been examined.²⁴ For a logarithmically rough landscape φ at hand (resulting from the different boundary condition across the contact plane), an exact calculation yields $D = 3/2$.^{24,25} Thus, we expect $\omega \leq 1$ for $D = 3/2$, and $\omega < 1$ for $D < 3/2$. The total energy of the dislocation loop

$$E_{\text{loop}} = E_{\text{core}} - E_{\text{dis}} \sim \mu' \ell^{2-D} L^D - \gamma' L^{\omega(D)} \quad (6)$$

does not admit a stable solution with $L \gg \ell$ for large $\mu' \approx \mu$. Hence the Bragg glass is stable to the spontaneous formation and proliferation of large dislocation loops. The possibility of a *marginally* stable Bragg glass for weakly disorder sample was first suggested in Ref. 14, based on the assertions that $\Delta E \sim gL$ and $E_{\text{core}} \sim cL$, where g is the bare disorder strength which can be made arbitrarily small and c is a given number.²³ The above analysis indicates that the dislocation loops are much more *strongly suppressed* at low temperatures by the anomalously large core energy.

Next, we investigate quantitatively the extent of the stability regime for the Bragg glass phase of the layered system (2). We consider quasi two-dimensional in-plane fluctuations, i.e., on the *shortest* scale in the direction perpendicular to the layers. Analytically, we apply a variational treatment, with the variational Hamiltonian $\tilde{\mathcal{H}}$ obtained by replacing the interaction potential $V_j[\phi]$ in Eq. (2) by the quadratic form $\tilde{V}_j[\phi] = \tilde{\mu} \phi^2/2$, which describes an elastic (i.e., dislocation-free) coupling in the direction perpendicular to the layers. The parameter $\tilde{\mu}$ has the meaning of an effective shear modulus and may be determined self-consistently within the variational treatment. The minimization of the variational free energy with respect to $\tilde{\mu}$ yields the self-consistency equation

$$\tilde{\mu} = \mu \langle \cos[\phi_{j+1}(\mathbf{r}_{\parallel}) - \phi_j(\mathbf{r}_{\parallel})] \rangle_{\tilde{\mathcal{H}}}. \quad (7)$$

This is evaluated using a Gaussian approximation which can be justified²⁶ in a controlled fashion. $\langle \phi \phi \rangle_{\tilde{\mathcal{H}}}$ contains contributions from (i) the quasi-2D VG regime $\langle \phi \phi \rangle_{2D}$ which dominates for $\tilde{\mu} \approx 0$; (ii) the 3D VG regime $\langle \phi \phi \rangle_{3D}$; and (iii) thermal fluctuations on scales smaller than the correlation length ℓ for large $\tilde{\mu}$. Using Eq. (4) for $\langle \phi \phi \rangle_{3D}$, and using $\langle \phi \phi \rangle_{2D} = 2(1 + \alpha) \ln(L)$ [where $\alpha \approx (l_{\parallel} g_0 / K)^4$ from Refs. 14 and 17], the following results are obtained:²⁷ The self-consistency equation has a stable solution with nonzero shear modulus only for $\mu > \mu_c = \bar{c}^2 K / \ell^2$.²⁸ For $\mu < \mu_c$, the system ‘‘melts’’ into a stack of decoupled 2D VG’s, i.e., it forms a smectic phase,²⁹ distinguished from the Bragg glass by a vanishing shear modulus $\tilde{\mu} = 0$. The transition at $\mu = \mu_c$ is first order, with a discontinuous jump in $\tilde{\mu}$. Our variational calculation yields a prefactor $\bar{c} \approx 60$ which depends very weakly on temperatures, as long as we are away from the melting temperature of the pure system.²⁶ It is illustrative to express μ and K in terms of the correlation length of the anisotropic system (2) in the \perp direction, $\ell_{\perp} \equiv \sqrt{\mu l_{\perp}^2 / K} \ell$. The above stability condition then becomes $\ell_{\perp} > \bar{c} l_{\perp}$, which may be viewed as the disordered-analog of the Lindemann criterion.

Clearly, the layered model (2) we used so far has two limitations: Displacements are uniaxial, and dislocation loops occur only in planes parallel to the vortex layers. We expect the difference between one- and two-component displacements to be analogous to the difference between scalar and vector charges in a Coulomb gas representation. Thus, the scaling of the relevant energy scales of a dislocation loop, E_{dis} and E_{core} , should be unchanged. If we exclude vacancies and interstitials, a dislocation loop *always* has to lie within a single plane spanned by its Burgers vector and the applied field.³⁰ Therefore, the scaling argument we presented for the layered system can also be applied to study the stability of the Bragg glass in the more common experimental situation of flux lines perpendicular to the CuO planes.

Naive generalization of our results to the isotropic system yields $\ell > c \cdot l$. We expect a larger numerical factor $c \geq \bar{c}$ since the two component system is less stable. Further taking into account of the ‘‘random manifold’’ regime which occurs on intermediate scales between l and ℓ ,^{14,22} we find²⁶ a *reduction* in the dislocation loop core energy, $E_{\text{core}} \sim (c_{44} c_{66})^{1/2} (l/\ell)^{2-2\zeta} \ell^2 L$, for a nonfractal dislocation loop of extent L . Here, c_{44} and c_{66} are the tilt and shear moduli, and $\zeta \approx 0.2$ is the roughness exponent characterizing the random manifold. Assuming that even a nonfractal dislocation loop can make the maximum energy gain $E_{\text{dis}} \sim \Delta E \sim (c_{44} c_{66})^{1/2} l^2 L$, we obtain the criterion $\ell > c^{1/2\zeta} l$. The increased numerical factor $c^{1/2\zeta}$ indicates a *reduced* stability regime for the Bragg glass, resulting from the reduced core energy. In the dense limit $\lambda \gg l$, if $\ell \geq \lambda$, we find that E_{core} is further reduced by a factor $(l/\lambda)^{2\zeta}$ leading to the criterion $\ell > c^{1/2\zeta} \lambda$, where the vortex spacing l is replaced by the range of the magnetic interaction λ . The Bragg glass cannot be stable in the limit $\lambda \rightarrow \infty$ (or at scales below λ for finite λ 's) because the long-ranged magnetic interaction gives rise to a much stronger disorder energy, $\Delta E \sim L^2$,¹⁴ which always exceeds E_{core} .

We can summarize these findings in the Lindemann-like criterion for the stability of the Bragg glass

$$\ell > c^{1/2} \zeta \max(l, \lambda), \quad (8)$$

with $c \gtrsim \mathcal{O}(50)$. In fact, the criterion (8) is equivalent³² to the more conventional Lindemann-criterion proposed recently by Ertaş and Nelson,³¹ with a corresponding Lindemann-number $c_L \approx 1/\sqrt{c}$. Our estimate of c yields $c_L \approx 0.15$, which agrees well with the experimentally accepted value. Using material parameters similar to Ref. 31 for a typical BSCCO, the criterion (8) yields an upper critical field $B_c \approx 500G$ below which the Bragg glass phase is stable.³² This coincides with the critical field where Bragg peaks appear in BSCCO (Ref. 12) and suggests that the observed Bragg peaks may

indeed be a manifestation of the Bragg glass. For fields exceeding B_c , the system is dominated by *strong disorders*, and its properties are not known. It may simply *melt* into a viscous line liquid, or it may form another type of vortex glass.¹⁴

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²⁷Qualitatively similar results are obtained using $\overline{\langle \phi \phi \rangle}_{2D} \sim \ln^2(L)$ as found in Ref. 16, provided $K \gg K_c$.
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