

Competitive Localization of Vortex Lines and Interacting Bosons

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We present a theory for the localization of three-dimensional vortex lines or two-dimensional bosons with a short-ranged repulsive interaction which are competing for a *single* columnar defect or potential well. For two vortices we use a necklace model approach to find a new kind of delocalization transition between two different states with a single bound particle. This *exchange-delocalization* transition is characterized by the onset of vortex exchange on the defect for sufficiently weak vortex-vortex repulsion or sufficiently weak binding energy corresponding to high temperature. We calculate the transition point and order of the exchange-delocalization transition. A generalization of this transition to an arbitrary vortex number is proposed.

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Introduction.—Melting of the Bose glass, the low-temperature vortex phase in type-II superconductors with columnar defects, remains a subject of constant interest [1,2]. The interest is motivated not only by the appeal and importance of understanding a basic phase transition of the vortex system, but, since the 3D vortex array is equivalent to a quantum 2D Bose system [3], vortex melting also offers a unique experimentally accessible model to explore the interplay between disorder and interactions in the delocalization transition of the corresponding strongly correlated 2D quantum system.

Numerous experimental observations on Bose glass melting (see, for example, Refs. [2,4]) revealed a characteristic kink in the low-field segment of the melting line suggesting a change of the melting mechanism. A recent experimental study of BSCCO samples with a *very low* density of columnar pins [1] allowed one to identify the low-field part of the melting curve as a depinning transition from a single columnar defect driven by vortex-vortex interactions. A theoretical study of the interacting boson system with a low-density of strong defects [5] demonstrated the possibility of an intermediate superfluid state where condensate and localized bosons coexist. Furthermore, it was found in Ref. [5] that interactions suppress localization and the increase of the boson density results in a sharp *delocalization crossover* into a state where all bosons are delocalized.

The model that is conventionally used in studies of quantum localization can be viewed as an ensemble of (interacting) particles immersed in a random field that can, in principle, localize or accommodate all particles; in other words, there is a dense array of pinning centers struggling to localize dilute, but interacting, particles. Reference [5] proposed to take an alternative approach and consider quantum particles or vortex lines of high-density competing for dilute traps or pinning sites. In this Letter, we extend this approach and explore the regime of low particle densities, i.e., a *finite* number of interacting

quantum particles. A related quantum system that has been studied previously are two electrons interacting in the region of a short-range attractive potential [6].

In this Letter, we investigate the formation of bound states in an ensemble of N strongly repulsive particles competing for a single attractive potential well. In the related vortex system, this corresponds to N vortices competing for a single columnar defect, i.e., to a physical situation where vortices outnumber columnar defects (magnetic fields $B = NB_\Phi$ well exceed the matching field $B_\Phi = \Phi_0/a_D^2$ where a_D is the average defect spacing and Φ_0 the flux quantum). We focus on the situation where mutual repulsion is strong enough to suppress double occupancy of the potential well and consider transversal dimensionalities $d \leq 2$, for which a bound state for a single particle in a symmetric potential well always exists (in the following, we use the language of either particles or vortex lines at our will). The main finding of this Letter is a new kind of delocalization phase transition driven by the *exchange* of the single bound particle with the $N - 1$ unbound ones. We first derive this result for $N = 2$ and then propose a generalization for arbitrary N . Finally, we obtain the transition temperature T_{de} for the exchange-delocalization transition in the vortex system and discuss the resulting phase diagram Fig. 2. Contrary to Ref. [5] where the boson (vortex) density was finite, we deal in this work with the genuine thermodynamic limit of *infinite* system size but *finite* particle (vortex) number N corresponding to $B, B_\Phi \approx 0$ with $N = B/B_\Phi$ finite. Thus the exchange-delocalization phase transition emerges in the limit $B \rightarrow 0$ and replaces the crossover that was found in Ref. [5] for macroscopic vortex density $B > 0$.

The model.—We describe a single vortex line in a sample of thickness L interacting with an attractive columnar defect by the Hamiltonian

$$\mathcal{H}_1[\mathbf{r}(z)] = \int_0^L dz \left\{ \frac{1}{2} \varepsilon_l (\partial_z \mathbf{r})^2 + V_D(\mathbf{r}(z)) \right\}, \quad (1)$$

where $(z, \mathbf{r}(z))$ is the vortex trajectory in $1 + d$ dimensions, the magnetic field aligned with the z axis, and ε_l is the stiffness or tilt modulus of a single vortex line; in an anisotropic superconductor $\varepsilon_l \approx \varepsilon^2 \varepsilon_0 \ln \kappa$ where $\varepsilon_0 = (\Phi_0/4\pi\lambda)^2$ is the characteristic vortex line energy, λ is the magnetic penetration depth, $\kappa = \lambda/\xi \gg 1$, ξ is the coherence length, and ε is the anisotropy parameter. $V_D(r)$ is the pinning potential from a single columnar defect positioned at $\mathbf{r} = 0$. $V_D(r)$ falls off exponentially for $r > \lambda$ [7] such that the large scale behavior of pinned vortex lines is well described using a cylindrical pinning potential well

$$V_D(\mathbf{r}) = U_D \quad \text{for } r < b_D, \quad V_D(\mathbf{r}) = 0 \quad \text{for } r > b_D \quad (2)$$

with a potential depth $U_D \equiv -\varepsilon_0 r_D^2/4\xi^2$ and an effective radius $b_D \equiv \sqrt{2\xi^2 + r_D^2}$ where r_D is the radius of the columnar defect [3].

In $d \leq 2$ dimensions the line is *always* bound to the defect as can be seen via mapping onto the ground-state problem of a quantum particle in a potential $V_D(\mathbf{r})$ for large L [8]. Choosing the energy scale such that the unbound vortex line has a free energy $E_0 = 0$, the free energy per length $E_1 < 0$ of the bound vortex line is obtained as the ground-state energy E_1 of the Schrödinger equation

$$[(T^2/2\varepsilon_l)\nabla_{\mathbf{r}}^2 - V_D(\mathbf{r})]\psi(\mathbf{r}) = -E_1\psi(\mathbf{r}), \quad (3)$$

$$E_1 = U_D f(T/T^*), \quad \text{where } T^* \equiv b_D \sqrt{\varepsilon_l |U_D|} \quad (4)$$

is a characteristic crossover energy, and $f(x)$ a scaling function. For $d = 2$ and the pinning potential (2), it has the asymptotic behavior $f(x) \approx 1 - \mathcal{O}(x^2)$ for $x \ll 1$ and $f(x) \approx x^2 \exp(-2x^2)/2$ for $x \gg 1$ [3].

For $N > 1$ we add the repulsive vortex interactions [3,9]

$$\mathcal{H}_N = \sum_{i=1}^N \mathcal{H}_1[\mathbf{r}_i(z)] + \sum_{i \neq j=1}^N \int_0^L dz V[|\mathbf{r}_i(z) - \mathbf{r}_j(z)|], \quad (5)$$

where $V(r) = 2\varepsilon_0 K_0(r/\lambda)$, and K_0 is the Bessel function. Double occupancy of the defect over an extended length is energetically disfavored if $\varepsilon_0 \gg |E_1|$, which is always the case at high enough temperatures. We also focus on the regime of large vortex spacing $a \gg \lambda$.

Exchange-delocalization transition for $N = 2$.—We investigate the localization behavior for the case $N = 2$ making use of a *necklace model* approach [10]. As the double occupancy of the defect is energetically unfavorable, the accessible configurations of the vortex lines consist of a necklacelike succession of two possible configurations *A* and *B*; see Fig. 1. In configuration *A*, line 1 is bound to the defect with the binding free energy $E_1 < 0$ and the transversal localization length $\xi_{\perp} = T/\sqrt{|E_1|\varepsilon_l}$, whereas line 2 is essentially in the unbound

state with the free energy $E_0 = 0$ and experiences rare collisions with the bound line 1. As we assume $\lambda \ll a$, the unbound line is exploring the region $r > \lambda$ of exponentially weak repulsion, whereas collisions occur at $r < \lambda$. Configuration *A* ends in an exchange event where the end points of the unbound line 2 attach to the defect again; see Fig. 1. At these exchange points configuration *A* can connect to configuration *B* where the roles of the particles are exchanged.

First, we estimate the energy cost of the localized collision and exchange events. In the presence of the repulsion (5), each return of line 2 to the bound line 1 costs an additional collision repulsion energy E_r which is determined by optimizing the sum of elastic and repulsive energies for a contact of length ℓ_r over which the typical line spacing is of the order $|\Delta\mathbf{r}| \approx \lambda$, $E_r \approx \varepsilon_l \lambda^2/\ell_r + \ell_r \varepsilon_0$:

$$\ell_r \approx \lambda \sqrt{\varepsilon_l/\varepsilon_0}, \quad E_r \approx \lambda \sqrt{\varepsilon_l \varepsilon_0}, \quad (6)$$

and $v_r \equiv \exp(-E_r/T) < 1$ defines the Boltzmann factor associated with each collision. Similarly, we estimate the energy cost E_{ex} of a localized exchange by optimizing the sum of the elastic energy and the loss of binding energy $|E_1|\ell_{\text{ex}}$ for a contact of length ℓ_{ex} over which the typical line spacing is of the order $|\Delta\mathbf{r}| \approx \lambda$, $E_{\text{ex}} \approx \varepsilon_l \lambda^2/\ell_{\text{ex}} + \ell_c |E_1|$. This gives

$$\ell_{\text{ex}} \approx \lambda \sqrt{\varepsilon_l/|E_1|}, \quad E_{\text{ex}} \approx \lambda \sqrt{\varepsilon_l |E_1|}, \quad (7)$$

and $v_{\text{ex}} \equiv \exp(-E_{\text{ex}}/T) < 1$ is the Boltzmann factor associated with each localized exchange.

Now we address the statistical mechanics problem of summing over all vortex line configurations. Adopting a coarse-grained description focusing on scales $r \gtrsim \lambda$ for transversal vortex fluctuations, we discretize the vortex system into segments of length $l = \lambda^2 \varepsilon_l/T$ in the z direction. In what follows, we calculate the grand-canonical partition sum $G(z) = \sum_n Z(n) z^n$ where $Z(n)$ is the partition sum for a system of length $L = nl$ and z is the fugacity. $\tilde{G}(E) = G(\exp(IE/T))$ is the Green's function at energy E for the corresponding two-particle quantum problem. The free energy density f of the system is determined by the real singularity z_0 of $G(z)$ closest to the origin by the relation $z_0 = \exp(lf/T)$. If $G_A(z)$ and $G_B(z)$ are the parti-

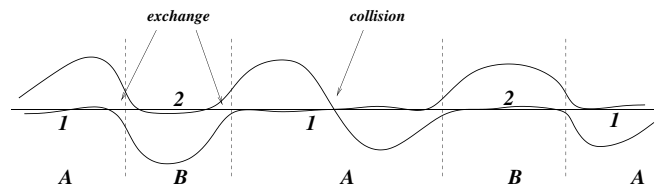


FIG. 1. Two particles binding alternately to a single columnar defect. The particle binding to the defect can be exchanged in localized events, and there are rare collisions between the free and the bound particle.

tion sums for configurations A and B , respectively, the full partition sum is obtained by summing over all alternating configurations G_A , G_B , $G_A v_{\text{ex}} G_B$, $G_B v_{\text{ex}} G_A$, $G_A v_{\text{ex}} G_B v_{\text{ex}} G_A$, \dots , separated by particle exchanges with the Boltzmann factor v_{ex} . Summing up the resulting geometric series we obtain

$$G(z) = \frac{G_A + G_B + 2v_{\text{ex}} G_A G_B}{1 - v_{\text{ex}}^2 G_A G_B} \Big|_z = \frac{2G_A(z)}{1 - v_{\text{ex}} G_A(z)}, \quad (8)$$

where we used $G_A = G_B$ because both configurations are related by a mere particle exchange. [Boundary effects are irrelevant; we allow either A or B at the ends of the defect in (8)]. According to Eq. (8), the singularity determining the free energy of the system is given either by the singularity of $G_A(z)$ corresponding to the state where the *same* line is always bound or by the solution of $1 = v_{\text{ex}} G_A(z)$ corresponding to alternating bound particles. The *exchange-delocalization transition* between these two states occurs if both singularities occur at the same value of z .

To move further, we calculate the grand-canonical partition sum $G_A(z)$. In the absence of the interline repulsion, the canonical partition sum in configuration A , $Z_A(n) = Z_1(n)Z_2(n)$, is a product of the partition sum of the bound line 1, $Z_1(n) = \exp(-nIE_1/T)$ [see Eq. (4)], and of the free line 2, which is attached with its end points to the defect. This restriction leads to $Z_2(n) = p_n \exp(-nIE_0/T)$ where p_n is the probability for the return of the unbound line to the defect and $E_0 = 0$ is its free energy per unit length. In the absence of the repulsion this return probability is given by the return probability of a random walk, $p_n \approx n^{-d/2}$, for large n . Then the partition function $G_A(z)$ is related to the generating function $P(z) = \sum_n p_n z^n$ for these return probabilities by $G_A(z) = P(wz)$ where $w \equiv \exp(-IE_1/T) = \exp(-\lambda \varepsilon_l E_1/T^2)$. For $1 - z \ll 1$, $P(z) \sim (1 - z)^{d/2 - 1}$ for $d < 2$ and $P(z) \sim -\ln(1 - z)$ for $d = 2$. Including the Boltzmann factor v_r in the random walk of the unbound line for each collision with the repulsive bound line localized at the defect [see Eq. (6)] leads to a modified generating function $P_r(z)$:

$$P_r(z) = P(z) + P(z)v_r P_r(z) - P(z)P_r(z). \quad (9)$$

In this relation the contributions from the repulsion-free walks with the Boltzmann factor 1 are subtracted, and the corresponding term with the modified Boltzmann factor v_r is added on the right hand side recursively for each collision. With this modification due to collisions, we finally obtain

$$G_A(z) = P_r(wz) = \frac{P(wz)}{1 + (1 - v_r)P(wz)}, \quad w \equiv e^{-IE_1/T} \quad (10)$$

for the grand-canonical partition function. $G_A(z)$ has a singularity at $z = 1/w$ corresponding to a free energy per length $f = E_1$ identical to that of a single bound particle

because the second unbound particle has $E_0 = 0$. The function $P(wz)$ diverges upon approaching $z = 1/w$, and, thus, we find $G_A(1/w) = P_r(1) = (1 - v_r)^{-1}$ at the singularity.

Now we turn to the exchange-delocalization transition determined by the singularities of (8). We have already found that the singularity of $G_A(z)$ (at $z = 1/w$ corresponding to the free energy $f = E_1$) describes, indeed, a single bound particle, i.e., a state always with the *same* line bound. We have argued above that there can be a real singularity closer to the real axis which is given by the solution of the equation $1 = v_{\text{ex}} G_A(z)$ and which corresponds to the phase with an exchanging bound particle. From the functional form (10) of $G_A(z)$, one readily verifies that this singularity is, indeed, the one closer to the origin and therefore represents the thermodynamically stable phase provided

$$v_{\text{ex}} \geq v_{\text{ex},c} = G_A^{-1}(1/w) = 1 - v_r. \quad (11)$$

According to (11), the exchange-delocalization phase transition occurs at the critical temperature T_{de} that is obtained from $\exp(-E_r/T_{\text{de}}) \approx E_{\text{ex}}/T_{\text{de}}$ where we assumed that $E_{\text{ex}} \ll T_{\text{de}}$ because $T_{\text{de}} > T^*$. Using estimates (6) and (7), the asymptotics of the function $f(x)$ for $d = 2$, and $\lambda/b_D = \kappa/\sqrt{2}$, we arrive at

$$T_{\text{de}} \approx T^* \kappa^{1/3} \quad (12)$$

for the delocalization transition temperature T_{de} in the vortex system. The transition takes place in the regime $T > T^*$ where a single vortex is only weakly bound to the defect [3]; see Fig. 2. Note that both Eqs. (4) and (12) are self-consistent equations for T^* and T_{de} , respectively, due to the temperature dependence of ξ and λ . Furthermore, it can be shown by expanding the equation $1 = v_{\text{ex}} G_A(z)$ about the transition point that the exchange-delocalization transition is continuous for all $d \leq 2$ and of infinite order for $d = 2$.

We expect our results to apply to all short-ranged potential wells and particle interactions that decay faster than $1/r^2$ for large separations r [11]. More realistic pinning potentials contain an intermediate $1/r^2$ behavior on scales

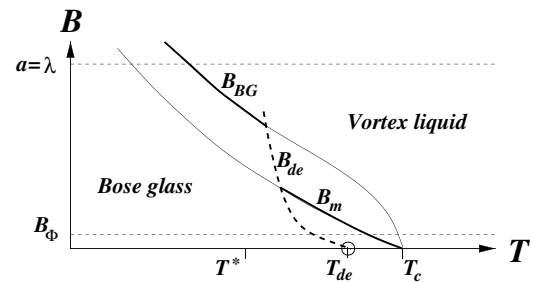


FIG. 2. Schematic phase diagram in the B - T plane. B_m is the pristine melting line, B_{BG} the Bose glass melting line, and B_{de} the exchange-delocalization crossover line which terminates at $B = 0$ in a genuine phase transition at temperature T_{de} (circle).

$\xi \ll r \ll \lambda$ [5,9], which slightly changes the function $f(x)$ and thus the exact value of T_{de} but not the universal properties of the delocalization transition.

General N .—We start from the exchange-delocalized state where all N vortices share the defect and consider the instability with respect to the exclusion of one of the vortices from the exchange. To this end we introduce N states (analogously to the states A and B for $N = 2$), where one vortex is unbound, i.e., excluded from the exchange, whereas the other $N - 1$ vortices share the defect. Then the necklace is a succession of possible states $i = 1, \dots, N$ each of which has a grand-canonical partition sum $G_i(z) = P(w_N z)/(1 + [1 - v_r(N)]P(w_N z))$ where $v_r(N) \equiv \exp(-\sqrt{N-1}E_r/T)$ is the Boltzmann factor due to the enhanced repulsion from the $N - 1$ vortices sharing the columnar defect. Similarly, $w_N \equiv \exp(-lf_{N-1}/T)$ is the Boltzmann factor for the binding free energy of the $N - 1$ vortices sharing the defect, which we approximate by $f_{N-1} \approx E_1$ or $w_N \approx w = \exp(-lE_1/T)$. Considering the generalized exchange between these N states and noting that the exchange of the single bound particle is associated with the Boltzmann factor v_{ex} , we arrive at the generalization of (8):

$$G(z) = \frac{NG_i(z)}{1 - (N-1)v_{ex}G_i(z)}.$$

The transition point for the exclusion of one particle from the exchange is given by the relation $v_{ex,c}(N) = [1 - v_r(N)]/(N - 1)$. As $v_{ex,c}(N)$ decreases for increasing N , also states with $N - 1$ or fewer exchanging particles become unstable for $v_{ex} < v_{ex,c}(N)$. This leads to the conclusion that particle exchange entirely stops at this point, and we thus identify $v_{ex} = v_{ex,c}(N)$ as the exchange-delocalization transition point of the N -particle system. For $N = 2$ our result reduces to Eqs. (11) and (12), whereas we find

$$T_{de}(N) \approx \begin{cases} T^* \ln^{1/2}(\kappa/\ln N), & \text{for } \ln N < \kappa, \\ T^* \kappa/\ln N, & \text{for } \ln N > \kappa \end{cases} \quad (13)$$

for large $N = B/B_\Phi$. Note that our approach is limited to the regime of large vortex spacing $a \gg \lambda$ or $N \ll a_D^2/\lambda^2$. The order of the transition is the same as for $N = 2$; i.e., the transition is of infinite order for $d = 2$.

Phase diagram.—In real vortex systems our results hold for the limit $B_\Phi \approx 0$. This implies also a vanishing vortex density $B \approx 0$ if $N = B/B_\Phi$ is fixed. A finite vortex density $\rho > 0$ corresponds to a system of the finite size $\propto 1/\rho^{1/2}$ which has no genuine phase transition. We thus conclude that the delocalization crossover line of Ref. [5] terminates in the genuine exchange-delocalization transition point at $B \approx 0$, which is given by Eqs. (12) or (13). For macroscopically large N the approximation of localized, well-separated exchange and repulsion events breaks down and our low-density approach becomes invalid, whereas the description by a condensate of bosonic particles used in

Ref. [5] works increasingly well in the high-density regime. The exact form of the crossover between both descriptions is an open question. In both descriptions the delocalization line $B_{de}(T)$ drops exponentially with temperature [see (13)] such that it intersects with both the pristine melting line $B_m(T)$ and the Bose glass melting line $B_{BG}(T)$; see Fig. 2. Beyond the delocalization line, the vortex line exchange at the defects sets in, which leads to line wandering and a liquidlike behavior even in the presence of columnar defects, which become irrelevant. Therefore, the relevant melting line is the pristine melting line for $T > T_{de}(B)$ in the delocalized phase, whereas it is the Bose glass melting line in the localized phase. Therefore, there exists a range of magnetic fields where the vortex lattice melts by undergoing the delocalization transition [5] as shown in Fig. 2. The resulting phase diagram is in good agreement with the experimental results regarding the melting of “porous” vortex matter [1].

Conclusion.—In conclusion, we have shown that the competitive localization of particles with mutual short-range repulsion by a short-range attractive defect leads to the existence of a genuine phase transition, the *exchange-delocalization transition*, which marks the onset of particle exchange at the defect. We have investigated the exchange-delocalization transition in a system of vortices with columnar defects or interacting bosons with localizing defects at low-density. We expect exchange-localization transitions to play an important role in various other systems where competitive localization occurs. In the introduction, we have already mentioned the quantum mechanical system of two interacting electrons competing for an attractive potential [6] that could be realized by a quantum dot. We also expect competitive localization to be relevant for biopolymers; for example, it applies to the competitive binding of two identical single strands of DNA to a single complementary DNA strand as it is important for DNA microarray engineering.

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